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The Functional Synthesis of Linear Plots

R. F. DRESSLER AND J. P. VINTI

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THE FUNCTIONAL SYNTHESIS OF LINEAR PLOTS

R. F. Dressler and J. P. Vinti[1]

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[1]National Bureau of Standards, Washington, D. C.

Abstract

In practical engineering or experimental work one often encounters a function F of many variables, F(x's, y's, z's), represented only by the families of curves obtained by plotting F against each of the x's on Cartesian graph paper, against each of the y's on semi-log paper, and against each of the z's on double-log paper. It frequently happens that these curves are all approximately straight lines over some limited range of interest. On the assumption that they are all true straight lines, the present note shows how to synthesize all the graphical representations for any number of parameters into the most general formula possible, expressing F as the product of a general multilinear function of the x's and the exponential of a constant-free multilinear function of the y's and of the log z's, with the coefficients in both multilinear functions being independent of the x's, y's, and z's.

1. INTRODUCTION

In a comprehensive investigation by one of the authors on assemblages of elastic shells, the various results for certain components of stress and displacement exhibited approximately linear behavior over the limited ranges of interest of the relevant parameters, when plotted on Cartesian graph paper, semi-log paper, or double-log paper. These results, derived from various numerical calculations and corroboratory experiments, are functions of many parameters, such as the various geometrical ratios defining shell shapes, the shell thickness, Poisson's ratio, the number of coupled shells, and other significant quantities. Rather than to retain these extensive results in the form of cumbersome families of graphs, it was desirable and useful to combine them in such a way as to obtain a single explicit formula for each dependent quantity, in terms of the above-mentioned independent parameters.

The problem of combining such results is, of course, not peculiar to investigations in elasticity, but often arises in experimental or engineering work of any nature. One can easily construct simple functions which will exhibit such "linear" properties, but the <u>most</u> <u>general</u> answer to the <u>inverse</u> problem is less obvious, especially when the number of parameters is large. Then, an unsystematic attempt to effect such a synthesis may prove infeasible or incomplete, and furnish no assurance that one has indeed constructed the most general function with these

properties. Our present note, therefore, formulates the general problem in a precise manner, derives its <u>most general</u> solution, and then illustrates this solution with some simple examples.

2. THE GENERAL PROBLEM

Consider a function F of the independent variables $x_1, x_2, \dots x_m; y_1, y_2 \dots y_p; z_1, z_2, \dots z_q$; and $u_1, u_2, \dots u_r$, such that a straight line results when we plot F against any x on ordinary Cartesian graph paper, against any y on semi-log paper, or against any z on double-log paper. Let the u's denote all other variables on which F may depend, but for which no such "linear" property exists. It is understood, of course, that in practice such linearity may hold only approximately, and only for a certain bounded range of the variable used as abscissa, and only for certain bounded ranges of the remaining variables, which appear as parameters. In this analysis, however, we relax these restrictions, assuming that each plot is a straight line for all values of abscissa and parameters.

We put

(1) $\ln z_1 = y_{p+1}, \ln z_2 = y_{p+2}, \dots \ln z_q = y_n$, $(n = p+q)$: then $F(x_1, \dots y_n, u)$ is a function such that the plot of F against any x or of lnF against any F is a straight line on Cartesian graph paper. Here u denotes the set $u_1, u_2, \dots u_r$.

Then

$$\ln F = A_k y_k + B_k, \qquad (k=1, 2, \dots n) \tag{2}$$

where A_k and B_k are functions of $x_1 \ldots x_m$, $y_1 \ldots y_{k-1}$, $y_{k+1}, \ldots y_n$, and of the u's. We then ask: if lnF is linear in each y when the other y's are all held constant, what is the most general form for lnF, as a function of the y's, that will represent such a property?

To answer this question, we first recall the definition of a <u>multilinear</u> function G, of several variables $t_1, t_2, \ldots t_s$, as a function which is the sum of a constant and a linear combination of all the products of the t's taken one at a time, two at a time,...s at a time, without repetition. For example, if s=3,

$$G(t_1, t_2, t_3) = a_o + a_1 t_1 + a_2 t_2 + a_3 t_3$$

$$+ b_1 t_2 t_3 + b_2 t_3 t_1 + b_3 t_1 t_2 + c t_1 t_2 t_3 \tag{3}$$

Any such multilinear function satisfies the differential equations

$$\partial^2 G / \partial t_k^2 = 0 \qquad (k=1, 2, \ldots, s) \tag{4}$$

(When the constant term vanishes, we term G a constant-free multilinear function.) It is then easily shown by induction that $G(t_1, t_2, \ldots, t_s)$ is the most general function of $t_1, t_2, \ldots, t_s$ which is linear in each t.

The Synthesis for the y's Alone:

On applying these considerations to (2), we find that

$$\ln F = g(x_1, x_2, \ldots, x_m, u) + N^{(x,u)}(y_1, y_2, \ldots, y_n), \tag{5}$$

where $N^{(x,u)}$ denotes a general constant-free multilinear function of the y's, with coefficients which are, a priori, functions of $x_1,\ldots,x_m$, u. On placing

(6) $$\exp g = f\,(x_1, x_2,\ldots x_m, u),$$

we obtain

(7) $$F = f(x_1, x_2,\ldots, x_m, u)\exp N^{(x,u)},$$

which gives the synthesis of the linearities of lnF versus the y's.

The Complete Synthesis:

Since F is linear in each x, it follows from the property of multilinear functions that

(8) $$F = P^{(y,u)}(x_1, x_2,\ldots,x_m),$$

where $P^{(y,u)}$ is a general multilinear function of the x's, the coefficients being functions of the y's and the u's. Then

(9) $$F = f(x_1, x_2,\ldots, x_m, u)\exp N^{(x,u)} = P^{(y,u)}\,(x_1, x_2,\ldots,x_m),$$

where by (4)

(10) $$\partial^2 N/\partial y_k^2 = 0 \qquad (k = 1, 2,\ldots,n)$$

(11) $$\partial^2 P/\partial x_j^2 = 0 \qquad (j = 1, 2,\ldots,m)$$

We next show that f is a multilinear function of the x's. To do so, put every y equal to zero in (9). Since N contains no constant term, it then vanishes, so that (9) becomes

(12) $$f(x_1, x_2,\ldots,x_m, u) = M^{(u)}(x_1, x_2,\ldots,x_m)$$

Here $M^{(u)}$ is simply the expression for $P^{(y,u)}$ with each coefficient evaluated at each y=0, so that it is a multilinear function of the x's with coefficients depending only upon the u's.

We now show that the coefficients in $N^{(x,u)}$ are independent of the x's, as follows. Insert (12) into (7), so that

$$F = M^{(u)} \exp N^{(x,u)} \tag{13}$$

and require that (13) satisfy (11). It follows that

$$\frac{\partial^2 M}{\partial x_j^2} + M\frac{\partial^2 N}{\partial x_j^2} + 2\frac{\partial M}{\partial x_j}\frac{\partial N}{\partial x_j} + M\left(\frac{\partial N}{\partial x_j}\right)^2 = 0, \tag{14}$$

where we have omitted the superscripts for convenience. Since M is multilinear in the x's, it follows from (4) that

$$\partial^2 M/\partial x_j^2 = 0, \tag{15}$$

so that

$$M\frac{\partial^2 N}{\partial x_j^2} + 2\frac{\partial M}{\partial x_j}\frac{\partial N}{\partial x_j} + M\left(\frac{\partial N}{\partial x_j}\right)^2 = 0 \tag{16}$$

If we now differentiate (16) twice with respect to y_k, we find with use of (10) that

$$2M\left(\frac{\partial^2 N}{\partial x_j \partial y_k}\right)^2 = 0, \tag{17}$$

whence

$$\frac{\partial}{\partial y_k}\left(\frac{\partial N}{\partial x_j}\right) = 0 \qquad \begin{array}{l} j = 1,2,\ldots,m \\ k = 1,2,\ldots,n \end{array} \tag{18}$$

Eq. (18) means that $\partial N/\partial x_j$ can be a function only of the x's and the u's. But N and $\partial N/\partial x_j$ are constant-free multilinear functions of the y's, with coefficients that are, a priori, functions of the x's and the u's. These results are compatible if and only if

$$\partial N/\partial x_j = 0 \qquad (j = 1, 2, \ldots m) \tag{19}$$

so that each coefficient in the constant-free multilinear function N must be independent of the x's.

We may thus rewrite (13) as

$$F = M^{(u)}(x_1, x_2, \ldots, x_m) \exp N^{(u)}(y_1, y_2, \ldots, y_n). \tag{2-}$$

When we return to the original formulation of the problem in terms of the x's, y's and z's, it follows that the <u>most general</u> functional form for F is given by

$$F = M^{(u)}(x_1, x_2, \ldots, x_m) \exp N^{(u)}(y_1, y_2, \ldots y_p, \ln z_1, \ln z_2, \ldots, \ln z_q), \tag{21}$$

where $M^{(u)}$ is a general multilinear function of the x's and $N^{(u)}$ a general constant-free multilinear function of the y's and ln z's, the coefficients in both being functions only of the u's.

3. SOME ELEMENTARY EXAMPLES

As short illustrations of the general result (21), we now give a few examples where F has linear plots against just three variables. In each case we list the specific form that (21)

assumes and the slopes and intercepts on the appropriate plots. By comparing the behavior of the slopes and intercepts in the various experimentally given families of curves that define the function F with these listed formulas, one can readily determine which coefficients vanish, if any, and thus obtain a specific formula for F in any actual case. Here "ln" denotes a natural logarithm and "log" a common logarithm. For an x or a y the intercept is taken at zero, while for a z it is taken at z = 1. For the logarithmic plots the slopes and intercepts are those of log F. In the following formulas, it is understood that the constants may be functions of the u's.

(a) $\boxed{x, y_1, y_2}$

$$F = (k_1x+k_2)\exp(a_1y_1+a_2y_2+by_1y_2)$$

Cartesian plot vs x:

Slope $S = k_1\exp(a_1y_1+a_2y_2+by_1y_2)$

Intercept $I = k_2\exp(a_1y_1+a_2y_2+by_1y_2)$

Semi-log plot vs y_1:

Slope $S = 0.4343(a_1+by_2)$

Intercept $I = \log(k_1x+k_2)+0.4343\ a_2y_2$

(b) $\boxed{x, y, z}$

$$F = (k_1x+k_2)\exp(a_1y+a_2\ln z+by \ln z)$$

$$= (k_1x+k_2)e^{a_1y} z^{a_2+by}$$

<u>Cartesian</u> plot vs x:

Slope $S = k_1e^{a_1y} z^{a_2+by}$

Intercept $I = k_2e^{a_1y} z^{a_2+by}$

<u>Semi-log</u> plot vs y:

Slope $S = 0.4343a_1+b \log z$

Intercept $I = \log(k_1x+k_2)+a_2\log z$

<u>Double-log</u> plot vs z:

Slope $S = a_2+by$

Intercept $I = \log(k_1x+k_2)+0.4343a_1y$

(c) $\boxed{y_1, y_2, z}$

$$F = k \exp(a_1y_1+a_2y_2+a_3\ln z+b_1y_1\ln z+b_3y_1y_2+cy_1y_2\ln z)$$

$$= k\, e^{a_1y_1+a_2y_2+b_3y_1y_2} z^{a_3+b_1y_1+b_2y_2+cy_1y_2}$$

Semi-log plot vs y_1:

Slope $S = 0.4343(a_1+b_3y_2)+(b_1+cy_2)\log z$

Intercept $I = \log k+0.434a_2y_2+(a_3+b_2y_2)\log z$

Double-log plot vs z:

Slope $S = a_3+b_1y_1+b_2y_2+cy_1y_2$

Intercept $I = \log k+0.4343(a_1y_1+a_2y_2+b_3y_1y_2)$

(d) $\boxed{y, z_1, z_2}$

$$F = k\exp(a_1y+a_2\ln z_1+a_3\ln z_2+b_1y\ln z_1+b_2y\ln z_2 +b_3\ln z_1\ln z_2+cy\ln z_1\ln z_2)$$

$$= ke^{a_1y} z_1^{a_2+b_1y} z_2^{a_3+b_2y} H,$$

where

$$H = z_1^{(b_3+cy)\ln z_2} = z_2^{(b_3+cy)\ln z_1}$$

Semi-log plot vs y:

Slope $S = 0.4343a_1+b_1\log z_1+b_2\log z_2+2.303c\log z_1\log z_2$

Intercept $I = \log k+a_2\log z_1+a_3\log z_2+2.303b_3\log z_1\log z_2$

Double-log plot vs z_1:

Slope $S = a_2+b_1y+2.303(b_3+cy)\log z_2$

Intercept $I = \log k+0.4343a_1y+(a_3+b_2y)\log z_2$

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